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TECHNICAL BULLETIN No. 13  
OREGON STATE HIGHWAY DEPARTMENT

R. H. BALDOCK  
Chief Engineer

# Rational Design Methods for Short-span Suspension Bridges for Modern Highway Loadings

By

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and

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*For City Council of  
Portland 10 Nov-10*

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OREGON STATE HIGHWAY COMMISSION  
SALEM

HENRY F. CABELL, Chairman  
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*(also 2 Photos)*  
July, 1940

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## FOREWORD

In a former publication by this department\* certain economic data pertinent to short-span suspension bridges were presented, the findings, in general, indicating that the suspension type, in many instances, constitutes a most effective and economic selection for short-span structures. In this connection, the writers wish to point out a few fundamental facts:

Suspension bridge designs, classified as to the method of stiffening the suspension system may be segregated into three groups: (1) unstiffened, (2) integrally stiffened, and (3) deck stiffened.

Structures in the first classification are not suitable for short-span highway bridges due to the fact that such structures are lacking in rigidity, and that the live loads produce excessive distortions, thus rendering the designs unsuitable for traffic.

Structures in the second class are suitable for highway traffic; however, this type has not enjoyed extensive use due principally to the excessive cost.

The deck-stiffened type has been used extensively, and has proved satisfactory. However, it possesses certain peculiarities which may at first mislead the designer when proportioning its various parts. This type comprises, as it were, a dual ensemble—a primary and a secondary part. The primary component is the suspension system (cables, hangers, and deck). Its function is to support the loads. It is essential to stability. The secondary component is the stiffening frame. Its purpose is to reduce those live-load distortions which, if not thus limited, would produce grade changes of a magnitude sufficient to render the structure unsuitable for heavy or high-speed highway traffic. The stiffening truss, in addition to its principal function, also operates to carry a certain proportion of the live

\* Oregon Highway Department Technical Bulletin No. 11.

load directly to the supports by virtue of its simple beam capacity. Since these two systems act simultaneously it is obvious that the stress distribution in the ensemble will not be determined from statics alone.

Stiffening frames may be either hinged or fixed at the towers, and the side spans may be either suspended or independently supported. Since space will not permit a consideration of all of these types, the present investigation has been limited to structures with *suspended side spans* and *two-hinged* stiffening frames.

As may be inferred from the title, this bulletin will be confined to suspension bridges of comparatively short-span (from 350 feet to 600 feet, main span length). In a subsequent bulletin under process of preparation at this time, the researches will be extended to longer spans, and an attempt made to develop certain design constants of universal application.

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order to simplify the analysis, the effect of the side spans will be neglected for the present, and the cable structure considered as fixed at the tower points "O" and "D".\*

The external loads acting on the cable consist of the dead load of the entire system (cables, stiffening frames, deck, etc.) which may be represented by the term "w" per unit length and the effect of the live loading which may be represented by the term "q" per unit length. The dead load "w" may be considered as uniform throughout the length of the span within the limits of accuracy desired. The live load "q" may or may not be considered as uniformly distributed, depending upon the degree of accuracy desired (this will be discussed later). It is, in fact, variable, as will be seen from Equation 8. In this connection it should be pointed out that the loading "q" does not represent the total live load but only that portion of the live load transmitted to the cable.

Let  $S_D$  represent the dead-load stress in the cable at any point;

$S_L$  represent the live-load stress in the cable at any point;

$\lambda$  represent the unit distortion of the cable under live load;

Then the average stress during the period of motion was  $S_D + \frac{1}{2}S_L$  and the total resilient energy stored within the cable is

$$W_1 = \int_0^l [S_D + \frac{1}{2}S_L] d\lambda \quad (1)$$

But, from Figure 2,

$$S_D = H_D \cdot \frac{ds}{dx} \text{ and } S_L = H_L \cdot \frac{ds}{dx}$$

$$\text{Also } d\lambda = S_L \cdot \frac{ds}{AE_c} = H_L \cdot \frac{ds}{dx} \cdot \frac{ds}{AE_c}$$

\* Side-span effects will be considered later.

$$W_1 = \int_0^l \left[ H_D \frac{ds}{dx} + \frac{1}{2} H_L \frac{ds}{dx} \right] d\lambda = \int_0^l \left[ H_D + \frac{H_L}{2} \right] \frac{ds}{dx} \cdot H_L \frac{ds}{dx} \frac{ds}{AE_c} = (H_D + \frac{H_L}{2}) \frac{H_L}{AE_c} \int_0^l \frac{ds^2}{dx^2} ds$$

Where:

$H_D$  = Horizontal stress in cable due to dead load;

$H_L$  = Horizontal stress in cable due to live load;

$$\lambda = \frac{S_L}{AE_c}$$

Whence

$$W_1 = [H_D + \frac{1}{2}H_L] \left( \frac{H_L}{AE_c} \right) \cdot \int_0^l \left( \frac{ds}{dx} \right)^2 ds \quad (2)$$

$$= [H_D + \frac{1}{2}H_L] \cdot \left( \frac{H_L}{AE_c} \right) \cdot L_E$$

Expressed in words, the internal energy stored within the cable equals the product of the dead load horizontal pull, plus one-half the live load horizontal pull, multiplied by the live load horizontal pull times the length integral

$$L_E = \int_0^l \left( \frac{ds}{dx} \right)^2 ds$$

divided by the area of the cable, times its modulus of elasticity.

Reasoning in a similar manner, the average external loading during the period of motion is represented by the term  $(w + \frac{1}{2}q)$ ; hence the external work which has generated the resilient energy represented by Equation 2 (and therefore must balance it) is represented by the expression

$$W_E = \int_0^l (w + \frac{1}{2}q) \cdot \Delta \cdot dx \quad (3)$$

Where  $\Delta$  represents the deflection of the cable at any point.

For elastic equilibrium, therefore

$$[H_D + \frac{1}{2}H_L] \left( \frac{H_L}{AE_c} \right) [L_E] = \int_0^l (w + \frac{1}{2}q) \cdot \Delta \cdot dx \quad (4)$$

\* The term  $L_E$  is the integral  $\int_0^l \left[ \frac{ds}{dx} \right]^2 ds$  which can be readily evaluated as

$L_E = H_D^2 [1 + 16\phi^2] + 3/32 \phi \log_e [4\phi + [1 + 16\phi^2]^{1/2}]$  ..... (2a)  
where  $\phi$  is the dead load sag ratio ..... f/l (see Figure 1).

The above equation contains the terms representing the horizontal component of cable stress for both dead and live loading; also the deflection  $\Delta$  at any point in the span and the term "q" representing that portion of the total live load transmitted from the stiffening frames to the cables. With these factors evaluated, the cable stress for the loadings assumed can be readily determined. Moreover, since the *residual* live load active against the stiffening system is obviously (p-q) per unit length between points "a" and "b" and (-q) per unit length over the balance of the span, the stiffening frame stresses also will be determinate.

The above derivation constitutes the basic theory of stiffened suspension bridges and is comparatively simple. Its application, however, is not as simple as the theory itself inasmuch as the evaluation of the various unknowns is a process somewhat involved.

Equation 4 contains four unknowns; viz:  $H_D$ ;  $H_L$ ; q and  $\Delta$ , and also the cable area "A" which obviously must be assumed for the solution, and subsequently corrected by a process of cut and try exactly as in the case of any other statically indeterminate frame. The term  $H_D$  is readily evaluated from statics

as  $\frac{wl^2}{8f}$  \* where "f" is the dead load sag at center line span. This

leaves for determination the terms q,  $\Delta$ , and  $H_L$ , each of which will be considered in order.

\* The expression  $H_D = \frac{wl^2}{8f}$  is mathematically correct only when the dead load

per unit length is constant throughout the entire length of span. A constant deck load and a constant cross section of cable produces a combined unit dead load of varying intensity increasing from the center toward the ends due to the inclination of the cable. This discrepancy, for a 900-foot main span, was calculated by the authors and found to be approximately one-tenth of one per cent. The error introduced by using this formula is, in most instances less than the discrepancy existing between the actual weight of the structure and the most accurate estimate the designer can make; moreover, the variation in the unit dead load caused by the inclination of the cable is in many cases partially offset by a corresponding variation

in the weight of the stiffening frame. The expression  $H_D = \frac{wl^2}{8f}$  can therefore

be used in designing any bridge of moderate length; however, excessively long-span structures should be investigated for this varying load.

$$-q = H_D \left[ 1 + \frac{H_L}{H_D} \right] \frac{d^2 \Delta}{dx^2} - w \left[ 1 + \frac{H_L}{H_D} \right] + w$$

Article 2—Evaluation of the Term "q" (representing that portion of the live load transmitted to the cable)

Consider any infinitesimal increment of cable length as a free body in equilibrium (Figure 3) under the action of dead load forces only.  $\Sigma v = 0$  yields the following:

$$H_D \frac{dy_2}{dx} - H_D \frac{dy_1}{dx} = -w dx$$

But

$$dy_2 - dy_1 = d^2 y$$

Whence

$$H_D \frac{d^2 y}{dx^2} = -w \quad (5)$$

In a similar manner, after the application of live load, we may write for elastic equilibrium:

$$[H_D + H_L] \frac{d^2 (y + \Delta)}{dx^2} = - (w + q) \quad (6)$$

Whence

$$-q = H_D \left[ 1 + \frac{H_L}{H_D} \right] \frac{d^2 \Delta}{dx^2} - \frac{H_L}{H_D} w \quad (7)$$

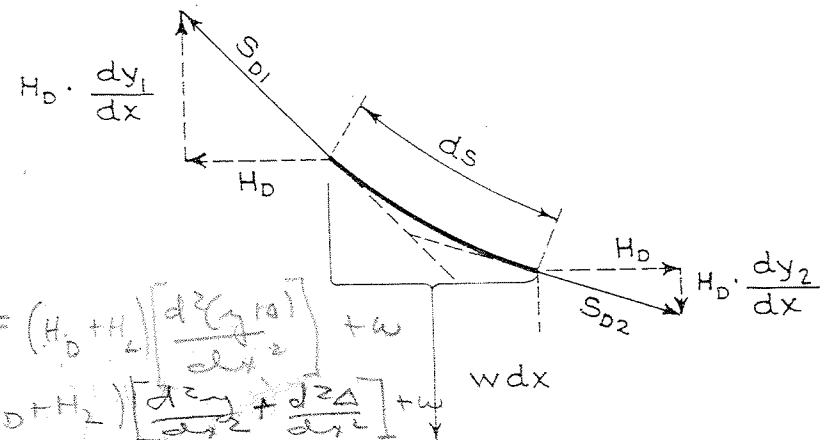


FIGURE 3

$$\begin{aligned} -q &= (H_D + H_L) \left[ \frac{d^2 (y + \Delta)}{dx^2} \right] + w \\ &= (H_D + H_L) \left[ \frac{d^2 y}{dx^2} + \frac{d^2 \Delta}{dx^2} \right] + w \\ &= (H_D + H_L) \left[ -\frac{w}{H_D} + \frac{d^2 \Delta}{dx^2} \right] + w = H_D \left[ 1 + \frac{H_L}{H_D} \right] \left[ -\frac{w}{H_D} + \frac{d^2 \Delta}{dx^2} \right] + w \\ &= H_D \left[ 1 + \frac{H_L}{H_D} \right] \left[ -\frac{w}{H_D} \right] + H_D \left[ 1 + \frac{H_L}{H_D} \right] \frac{d^2 \Delta}{dx^2} + w \end{aligned}$$

- I ALIGNMENT: DIRECTLY ABOVE EXISTING; LOWERED AFTER ERECTION AND DISMANTLEMENT OF EXISTING.
- II LENGTH: 1920' (RE 2000' EXISTING)
- III TYPE: DOUBLY SYMMETRIC SELF-ANCHORED SUSPENSION; 480', 960', 480' SPANS
- IV FUNCTIONS: TWO 16' AUTO LANES TOP  
ONE 10' RAIL LINE BOTTOM  
TWO 8' BICYCLE/PEDESTRIAN LANES ON EACH SIDE
- V TOWERS: 250' ABOVE PIERS
- VI CABLES: SAG RATIO  $1/6$  TO DECK'S CENTROID
- VII MATERIAL: A242 STEEL; 30 KSI WORKING STRESS
- VIII DESIGN METHODS: CLASSICAL ELASTIC  
McCULLOUGH'S "BETA"
- IX LOADINGS: DEAD 1610 lb/ft STRUCTURE  
(GLOBAL) DECK 2770 lb/ft } QUASI-LIVE  
RAILS 120 lb/ft }
- X LOADINGS: TWO 30,000 LB AXLE SETS PER LAKE  
(LOCAL) ONE 60,000 LB RAIL CAR  
100 LB/FT<sup>2</sup> BICYCLE-PEDESTRIANS
- XI PROPERTIES OF TRUSS  
VERTICAL MOMENT OF INERTIA: 78.0 FT<sup>4</sup>  
LATERAL MOMENT OF INERTIA: 71.2 FT<sup>4</sup>

## RADIUS OF GYRATION

9.13' VERTICAL; 9.19' LATERAL !

THEORETICAL EULER COLUMN LOAD  
FOR SELF-ANCHORED 1920' LENGTH.

$$K=1, L=960', r=9.16'$$

$$KL/r = 960'/9.16' = 105$$

$$F_E = K\pi^2 EI / L^2$$

$$= 3,325,000 \text{ LB} \quad (1,618 \text{ TONS})$$

## HORIZONTAL DEAD LOAD

$$H_D = w l^2 / 8f$$

$$w = \text{dead load/ft} = 4500 \text{ lb/ft}$$

$$l = \text{span} = 960 \text{ ft}$$

$$f = \text{sag} = 960/6 = 160 \text{ ft}$$

$$H_D = 4500 \times 960^2 / 8 \times 160$$

$$= 3,240,000 \text{ LB}$$

$$H_D < F_E \text{ AS REQUIRED!}$$

I STEEL DECK WEIGHS  $\approx 25 \text{ lb/ft}^2 + \text{FASTENERS}$   
 AERO FORCES ON SUSPENSION BRIDGES

II DEAD LOAD REDUCTION W/ STEEL DECK

A. KEEP STRINGER DESIGN FOR STIFFNESS

B.  $25 \text{ lb/ft}^2 \times 30' = 800 \text{ lb/ft}$

C. REDUCTION 1.  $\rightarrow 57.3\%$

1.  $4500 \text{ lb/ft} - 2773 + 800 = 2527 \text{ lb/ft}$

2.  $2527 / 4500 = 0.56 \text{ FACTOR}$

III HORIZONTAL DEAD LOAD IN CABLE

A.  $3,240,000 \text{ lb} \times .56 = 1,819,440 \text{ lb}$

B. FRACTION OF RULER LIMIT

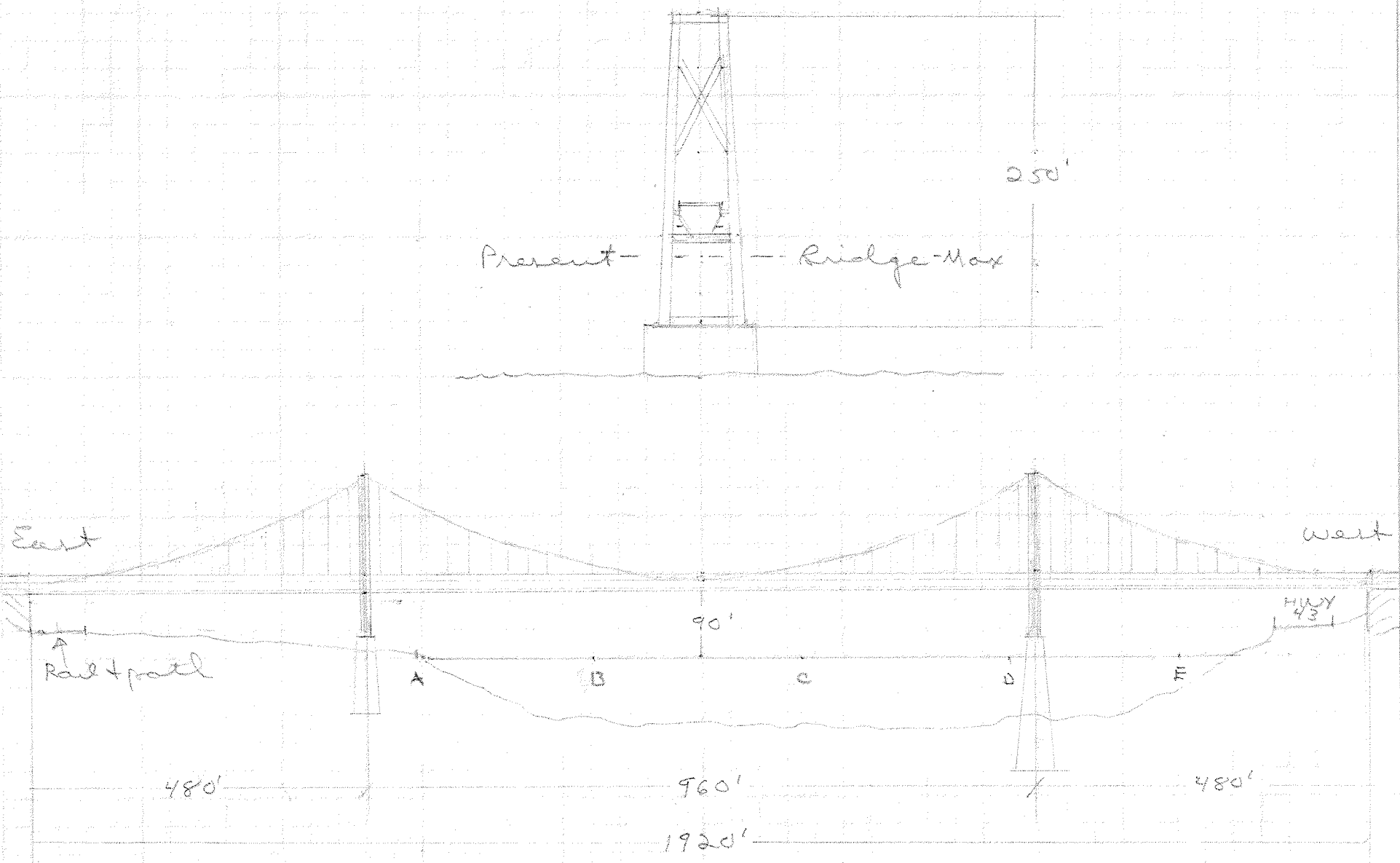
$1,819,440 / 3,325,000 = 0.55$

IV USE DIAGONAL STEEL DECKING

STEEL BRIDGE

DRILLING  
PROFILES

NO BRIDGE



A, B, C, D, E are locations of existing piers.

20  
40



I CF. CBM, #13, p 10.  $H_D = wL^2/8f$

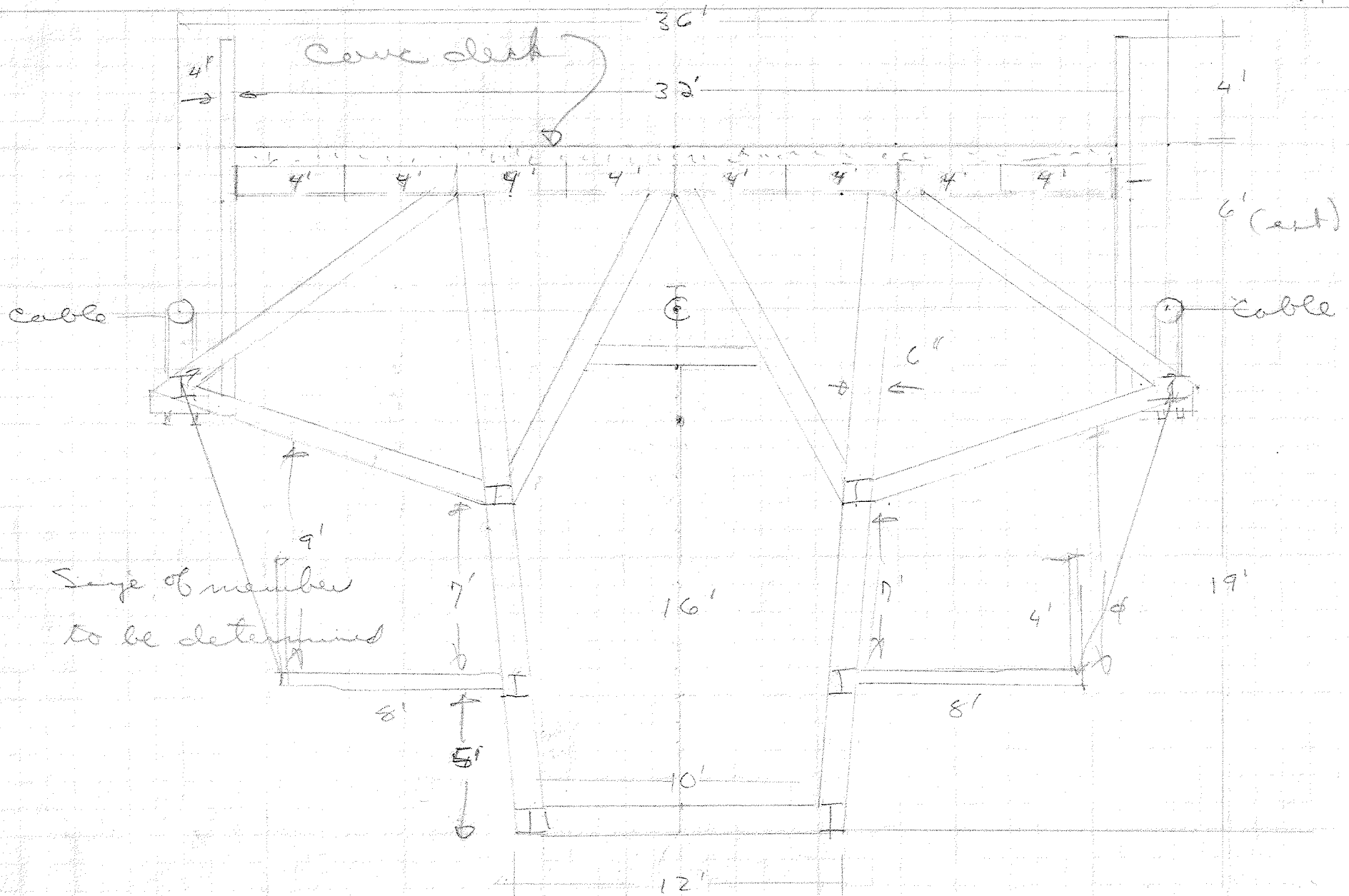
$w = \text{dead load} = 4500 \text{ lb/ft}$

$L = \text{span} = 960 \text{ ft}$

$f = \text{sag} = 960'/6 = 160'$

$H_D = 4,500 \times 960^2 / 8 \times 160$

$= 3,240,000 \text{ lb (1,620 TONS)}$



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## II MATERIAL: A242 "WEATHERING" STEEL

$$\sigma_y = 50 \text{ ksi}; \sigma_x = 30 \text{ ksi}; \sigma_z = 20 \text{ ksi}$$

$$(\sigma_x' = 0.60 \sigma_y = 30 \text{ ksi})$$

 AISC 6th  
5-91

## III ROAD DECK

A DECK: 8 in concrete on steel pan, 32 ft wide

1. DEAD:  $130 \text{ lb/ft} \times (8/12) \times 32' = 2773 \text{ lb/long ft}$

2. LIVE: 50 k/axle set in both lanes of 25 ft panel span, distributed among 3 stringers in each lane;  $50 \text{ k} \times 2/\text{panel} = 100 \text{ k/panel}$

3. (DEAD + LIVE)/panel =  $2773 \times 25' = 69,333 \text{ lb}$   
 $+ 100,000 \text{ lb}$

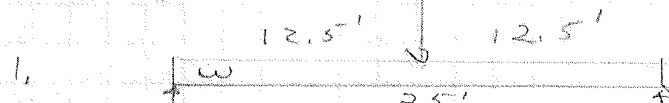
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 $169,333/\text{panel}$ 


---

B STRINGERS, 4' O.C.

← long →



$$w = (130 \text{ lb/ft}) (8/12) 4' = 347 \text{ lb/ft}$$

$$P = 50,000 \text{ lb}/3 \text{ stringers}$$

$$M_w = wL^2/8 = 347 \times 25^2/8$$

$$= 27,083 \text{ lb-ft}$$

$$M_p = PL/4 = (50,000/3)(25/4)$$

$$= 50,000 \times 25/12$$

$$= 104,167 \text{ lb-ft}$$

$$M_w + M_p = 131,250 \text{ lb-ft}$$

$$M_T = 1,575,000 \text{ lb-in}$$

FULL DEAD LOAD

$$4500 \text{ lb/ft (p9)}$$

$$- 2770 \text{ lb/ft Deck}$$

---

 $1730 \text{ lb/ft}$ 


---

$$- 120 \text{ lb/ft Rails}$$

$$1610 \text{ lb/ft TRUCK}$$

Dead load

EST TOTAL WEIGHT

$$1920' \times 4500 \text{ lb/ft}$$

$$= 8,640,000 \text{ lb}$$

 22-141 50 SHEETS  
 22-142 100 SHEETS  
 22-144 200 SHEETS

 CAMPBELL  
 22-141 50 SHEETS  
 22-142 100 SHEETS  
 22-144 200 SHEETS

 AISC 6th  
 2-120  
 122

II B 3

$$S_x > M_T / T_f$$

$$= 1,575,000 / 30,000$$

$$= 52.5 \text{ in}^3; 12 \text{ W } 75 \Rightarrow 58.2$$

There are effectively 8 of these  
 Add  $8 \times 45 = 360 \text{ lb/long-ft}$

Assume shear is ok

4. Net dead load/long-ft is now

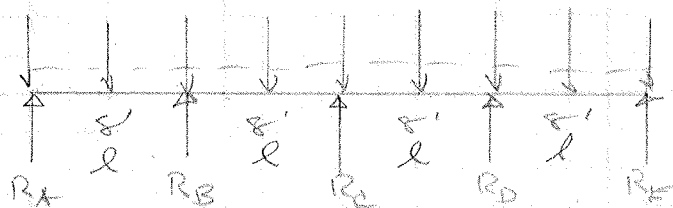
$$2773$$

$$+ 360$$

$$3133 \text{ lb/long-ft}$$

C Main cross girders, 32'

1. There are several different loads, but  
 width  $\rightarrow$



let us sum all live and dead loads  
 and compute an equivalent uniform  
 load for a 25' panel

i) Dead load of deck:  $(2773 \text{ lb/ft}) \times 25' = 69,325 \text{ lb}$

ii) Live load of 2 axle sets:  $59,000 \times 2 = 109,000 \text{ lb}$

iii) Total  $169,325 \text{ lb}$

iv) Equiv. uniform load

$$169,325 \text{ lb} / 32' = 5,291 \text{ lb/ft}$$

2. i)  $M_{\max} = 1/8 w l^2$

$$= 0.1701 \times (5291 \text{ lb/ft}) \times 8^2$$

$$= 57,604 \text{ lb-ft}$$

II B 2 i)  $M_{max} = 691,252 \text{ lb-in}$

ii)  $V_{max} = 0.607 w - l$

$$= 0.607 \times (5291 \text{ lb/ft}) \times 8$$

$$= 25,693 \text{ lb}$$

3.  $S_2 \geq 6921,252 \text{ lb-in} / (30,000 \text{ lb/in}^2)$

$$= 23.0 \text{ in}^3; 12B25 \Rightarrow 25.8 \text{ in}^3$$

4. Check shear

i) Web is  $12" \times 1/4" = 3 \text{ in}^2$

ii)  $25,693 \text{ lb} / 3 \text{ in}^2 = 8,564 \text{ lb/in}^2$

ok

$$< 20,000 \text{ lb/in}^2 = \tau_a$$

4 Net dead load per long ft is now  $3,133 \text{ lb/ft}$

$12B22 \times 32' \text{ width} / 25' \text{ panel}$

$$+ 28 \text{ lb/ft}$$

$$\underline{3,161 \text{ lb/long ft}}$$

#### IV RAIL DECK

A DEAD  $2 \times 6 \text{ in} / \text{yd} = 2 \times 20 \text{ lb/ft} = 40 \text{ lb/ft}$  RAILS

FISC 624

1-107

1. RAILS:  $2 \times 60 \text{ lb/yd} = 2 \times 20 \text{ lb/ft} = 40 \text{ lb/ft}$

2. TIES: CONCRETE;  $2' \text{ o.c.}; \approx 1 \text{ ft}^3 \times 150 \text{ lb/ft}^3$

$$(150 \text{ lb/tie}) / 2' \text{ o.c.} \approx 75 \text{ lb/ft}$$

3. RAILS + TIES

$$\underline{115 \text{ lb/long ft}}$$

B LIVE:  $60,000 \text{ lb}$  loaded car on 2 axle sets

1. axles of each set  $\approx 4'$  apart;  $60,000 \text{ lb} / 4 \text{ axles}$

$$= 15,000 \text{ lb/axle}$$

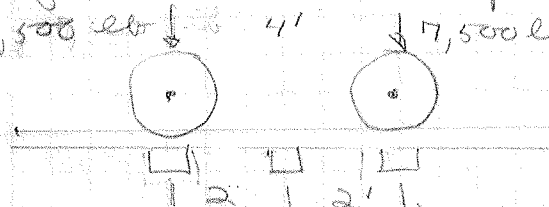
$$= 15,000 \text{ lb/axle}$$

$$(15,000 \text{ lb/axle}) /$$

$$(2 \text{ wheels/axle})$$

$$= 7,500 \text{ lb/wheel}$$

← long →



(4)

**PORTLAND CITY COUNCIL  
COMMUNICATION REQUEST  
Wednesday Council Meeting 9:30 AM**

Council Meeting Date: November 10, 2010

Today's Date October 8, 2010

AUDITOR 10/08/10 AM 10:27

Name JAMES B. LEE

Address 6016 SE MITCHELL, PORTLAND 97206

Telephone 503-771-6128 Email CADWAL@MACFORCEGO.COM

Reason for the request:

ECONOMIC DESIGN FOR

SKILLWOOD BRIDGE

James B. Lee  
(signed)

- Give your request to the Council Clerk's office by Thursday at 5:00 pm to sign up for the following Wednesday Meeting. Holiday deadline schedule is Wednesday at 5:00 pm. (See contact information below.)
- You will be placed on the Wednesday Agenda as a "Communication." Communications are the first item on the Agenda and are taken promptly at 9:30 a.m. A total of five Communications may be scheduled. Individuals must schedule their own Communication.
- You will have 3 minutes to speak and may also submit written testimony before or at the meeting.

*Thank you for being an active participant in your City government.*

**Contact Information:**

Karla Moore-Love, City Council Clerk  
1221 SW 4th Ave, Room 140  
Portland, OR 97204-1900  
(503) 823-4086 Fax (503) 823-4571  
email: [kmoore-love@ci.portland.or.us](mailto:kmoore-love@ci.portland.or.us)

Sue Parsons, Council Clerk Assistant  
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email: [sparsons@ci.portland.or.us](mailto:sparsons@ci.portland.or.us)

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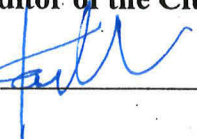
Request of James B. Lee to address Council regarding economic design for  
Sellwood Bridge (Communication)

NOV 10 2010

PLACED ON FILE

Filed NOV 05 2010

**LaVonne Griffin-Valade**  
Auditor of the City of Portland

By 

COMMISSIONERS VOTED AS FOLLOWS:		
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2. Fish		
3. Saltzman		
4. Leonard		
Adams		